Control of Pattern Formation in Excitable Media

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Abstract

A method for controlling spiral waves in excitable media is proposed. Applying suitable weak external forcing to the systems´s slow variable, we can successfully control spiral waves to a desired new patterns. The effectiveness of the control law is illustrated via numerical simulations on a 2D model.

Keywords: Spiral waves; Excitable media; Feedback control; Biological systems.

1 Introduction

A broad attention has been devoted from many years to the study of pattern formation in distributed systems, due to its importance in the fields of biology, chemistry, physics and ecology [1-3]. A particular interest is related to the case of reaction-diffusion equations in two-dimensional excitable systems, which can be used to model the electrical activity of biological tissues-nerve fibers [1], cardiac muscle [4,5], brain tissue [6] and to reproduce a lot of main phenomena experimentally observed [2,6,7].

In one-dimensional media, with parameters in the right range, excitable media respond to strong enough stimulation by propagating a soliton-like pulse at steady speed, with steady profile. Impulse propagation along nerve axons is a well-known example of this phenomenon [1,2]. In two-dimensional media spiral waves can emerge with both appropriate initial conditions and parameters [8,9]. Examples include waves of chemical activity in Belousov-Zhabotinsky (BZ) reaction [10], electrical activity in cardiac tissue [4,5], aggregation of starving slime mold amebae *Dictyostelium discoideum* [11], and catalytic reactions on platinum surface [12].

In some cases spiral waves and propagating pulses are undesirable because of their harmfulness. For example, spirals in cardiac muscle play an essential role in heart diseases such as arrhythmia ventricular fibrillation, the major reason behind sudden cardiac death [4,5]. Therefore, control of nonlinear waves in excitable media is of much practical interest. The control of pattern formation has attracted much attention not only in excitable media but also for the potential applications in other fields. For example, the method of directional quenching has been used to control the microphase separation of diblock copolymer [13]. It was also found that the geometric size of the system can be useful in controlling the formation of optical patterns [14]. Moreover, with the rapid development of nanotechnology, pattern formation now can be controlled at nanometer scale [15] resulting in new surface and bulk nanostructured materials with unique or superior properties.

Proposed methods for controlling nonlinear waves in excitable media comprised feedback [16-18] and non-feedback methods [19-20]. Recently, Osipov and Collins [19] have proposed a general mechanism for suppressing non-steady state motions – propagating pulses, spiral waves, spiral-waves chaos – in excitable media. This approach is based on two points: (1) excitable media are multistable, and (2) traveling waves in excitable media can be separated into fast and slow motions, which can be considered independently. Osipov and Collins [19] showed that weak impulses can be used to change the values of the slow variable at the front and back of a traveling wave, which leads to wavefront and waveback velocities that are different from each other. This effect can destabilize the traveling wave, resulting in

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a transition to the rest state.

In this work, we propose a model-based feedback control approach to control spiral waves arising in excitable media. We can successfully control spiral waves to a desired new patterns by using a small time-dependent external perturbation on the slow system variable. This work is organized as follows: In the next section, the model used in this study is described. The nonlinear dynamics arising is such a systems is also presented. In Section 3, the feedback control method is described. In section 4 we applied the proposed control method to control 2D excitable media. Finally, in Section 5 we close this work with some concluding remarks.

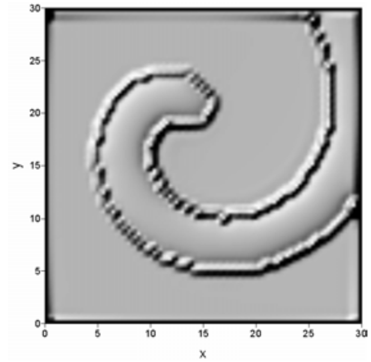

Figure 1: Two-dimensional phenomena in excitable media. Parameter values are given in text.

2 The Mathematical Model

Consider nonlinear reaction-diffusion systems describing an excitable media given by [2,9]:

$$\begin{aligned} \frac{\partial u}{\partial t} &= D_u \nabla^2 u + F(u,v) \\ \frac{\partial v}{\partial t} &= D_v \nabla^2 v + \varepsilon G(u,v) \end{aligned} \tag{1}$$

where u and v are the fast and slow variables. D_u is the diffusion coefficients for the fast variable and D_v is the diffusion coefficients for the slow variable. The excitability of a system may be defined by the inverse of ε. Upon increase of ε the ability of an excitable media to propagate waves usually is lost. The functions $F(u,v)$ and $G(u,v)$ express local kinetics of the variables u and v. In the following, without lost of generality we consider $D_v = 0$. In fact, this is the case for several biological systems since there are nondiffusing variables [3,8]. Two variable models of the above general form are very common in the study of excitable systems the Fitzhugh-Nagumo (FHN) model being the most famous example. Various models differ principally in the choice of the reaction terms *i.e.*, the functions F and G. Propagating waves of excitation are frequently found in excitable media such as BZ reagent [10] and in nerve cells and cardiac muscle [1,4]. In two space dimensions these waves commonly take the form of rotating spirals [9]. In three dimensions these waves can take quite exotic forms, but commonly the underlying spatial structure is that of a scroll [9].

Consider the 2D case as given in Barkley [8]. No-flux boundary conditions are imposed on the domain boundary. The following local kinetics was considered,

$$\begin{aligned} F(u,v) &= u(1-u)\left[u - \frac{(v+b)}{a}\right] \\ G(u,v) &= u - v \end{aligned} \tag{2}$$

The parameter values for the numerical simulation are: $a = 0.75$, $b = 0.01$, $1/\epsilon = 50$. An implicit scheme inspired by Barkley [8] is used here in order to integrate the equation (1) in the 2D case. Figure 2 shows a stable spiral wave generated with the above kinetics and parameters. The spiral wave shown in Figure 1 is stable in the sense that it is insensitive to small noise impacts and to the slight change of the initial condition and it persists forever unless some external forces drive the system away.

Since the appearance of propagating pulses and spiral waves in excitable and oscillatory media is often an undesirable effect, leading to unpredictable consequences for many applications [5, 12], there is a need to develop effective methods to suppress and control both propagating pulses and spiral waves. This will be addressed in the next section.

3 The Control Method

To introduce an external force that can be used for produced a weak external stimulus, Eq. (1) is changed by,

$$\begin{aligned} \frac{\partial u}{\partial t} &= D_u \nabla^2 u + F(u,v) \\ \frac{\partial v}{\partial t} &= \varepsilon G(u,v) + c \end{aligned} \tag{3}$$

where c is the control input in our feedback control law. The external control input c is a plausible manipulated variable since it is more readily amenable

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to experimental manipulation. For instance, a periodic, small amplitude, uniform electric field can be used to initialize directed drift of spiral wave in excitable media [16,18]. The light-sensitive variant of the BZ reaction constitutes an experimental system where the excitability of the medium can be manipulated by the intensity of incident light [10].

Since there exists high uncertainties in the rate constants and kinetic values and in order to not require much knowledge of terms that involves uncertain parameters, for control design purposes we define a modeling error function as follows,

$$\eta = \varepsilon G(u,v)$$

such that

$$\frac{\partial v}{\partial t} = \eta + c \tag{4}$$

Let v_r be a desired reference and $\overset{\bullet}{v_r}$ be the time-derivative of the desired reference signal. Consider the inverse-dynamics feedback function,

$$c = -[\eta + \tau_c^{-1}(v - v_r) + \overset{\bullet}{v_r}] \tag{5}$$

such that

$$\frac{\partial v}{\partial t} = \frac{\partial v_r}{\partial t} - \tau_c^{-1}(v - v_r) \tag{6}$$

It is noted that the dynamics (6) is stable and $v \to v_r$ asymptotically with τ_c as the mean convergence time. The closed-loop time constant τ_c is a control design parameter that can be chosen as the mean time of dominant excitable frequency. The above feedback function c can not be implemented just as it is because the modeling error signal η is unknown, then, the following observer is proposed to get the estimate signal $\overline{\eta}$,

$$\frac{\partial \overline{\eta}}{\partial t} = \tau_e^{-1}(\eta - \overline{\eta}) \tag{7}$$

where τ_e is the estimation time constant, which determines the smoothness of the modeling error estimation and can be chosen as $\tau_e < \frac{1}{2}\tau_c$. From (4), we know that $\eta = \partial v/\partial t - c$. Hence

$$\frac{\partial \overline{\eta}}{\partial t} = \tau_e^{-1}(\frac{\partial v}{\partial t} - c - \overline{\eta})$$

introduce the variable $w \overset{def}{=} \tau_e \overline{\eta} - v$. Then, the estimator (7) can be realized as follows:

$$\begin{aligned} \frac{\partial w}{\partial t} &= -c - \tau_e^{-1}(w + v) \\ \overline{\eta} &= \tau_e^{-1}(w + v) \end{aligned} \tag{8}$$

which is initialized as follows. Since η is unknown, we have that $\overline{\eta}_0 = 0$. Therefore, $w_0 = -v_0$.

The practical computed control law is then given by

$$c^r = -[\overline{\eta} + \tau_c^{-1}(v - v_r) + \overset{\bullet}{v_r}] \tag{9}$$

For consider possible physical restrictions in the magnitude of external stimulus we include a saturation function given by

$$c_{\mathrm{real}} = Sat(c_t^C) \tag{10}$$

where

$$Sat(c_t^C) = \begin{cases} c_{\min} & if \quad c \le c_{\min} \\ c & if \quad c_{\min} < c < c_{\max} \\ c_{\max} & if \quad c \ge c_{\max} \end{cases}$$

thus, the control input is limited by $c_{\min}$ for the minimum external signal and $c_{\max}$ for the maximum external signal for the application of weak pulses on a slow variable in excitable media. Positive values of c destabilize the nonlinear wave by decreasing the wave width, whereas negative values of c destabilize the wave by increasing the wave width [19]. The stability analysis of the closed-loop systems is beyond of the scope of this paper. However, this can be borrowed with stability arguments from singular perturbation theory and energy methods for distributed parameter systems [21].

4 Numerical Simulations

We have taken as a case studie the controlled transition from a stable spiral wave (Figure 2-a) to a new spatiotemporal pattern given in Figure 2-f. Color range form white at the minimum value of v and dark at the maximum value of v. A sequence of wave patterns after that the control law is activated is summarized from Fig. 2-c to 2-h, in which it is show the evolution from the initial pattern to the formation of a new spatiotemporal pattern. After some time, the original pattern faded away and is replaced by the reference pattern given in Figure 2-b. To illustrate the temporal evolution of the pattern, we shown the space-time diagram in Figure 3-a along the positions in x, y $(31,21)$, $(31,36)$ and $(31,51)$ as compared with the reference. Figure 3-b represents the evolution of the control inputs in the same positions as Figure 3-a associated to the field v. It can be seen from Figure 3-b that the control input no require much effort to drive the system to a new pattern given by the reference 3-b. Figure 4 shows the corresponding sequence of control inputs for the sequence of the v-field given in Figure 3. It can be seen

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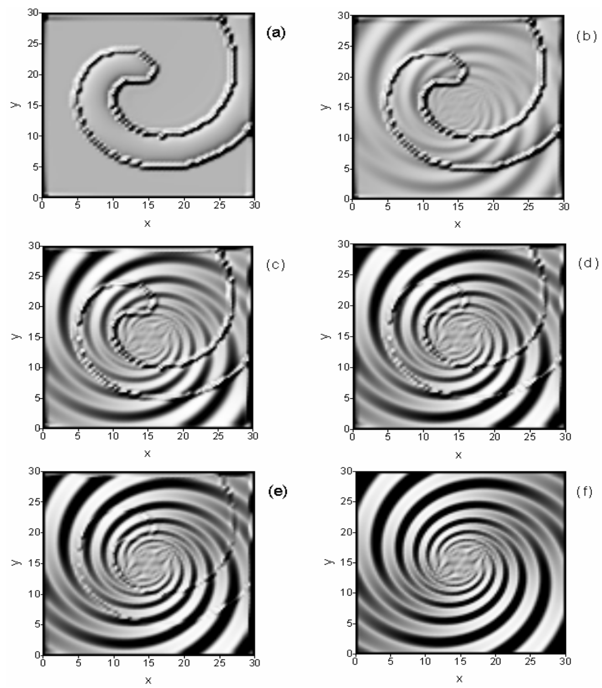

Figure 2: Sequence of wave patterns.

that in order to obtain a new pattern, the external control input evolves towards a spiral pattern.

Generally, an external injection into a small local region cannot essentially change the pattern of Figure 1, the spiral remains the same with slight deformation only. In realistic cases, it is important to suppress spiral waves by injecting the control action in localized zones, however, this is beyond of the scope of this paper and will be presented elsewhere. As described elsewhere [22], implantable cardioverter defibrillators offer therapy by delivering electrical stimulation directly to the heart using relatively simple algorithms. In contrast, a preferred therapy for a reentrant arrhythmia would exploit the dynamics of the arrhythmia and use appropriately timed low-voltage pulses or perhaps even a single pulse to collide with and annihilate nonlinear waves. In addition to the use of electrical stimulation to control arrhythmia, it is also well known that strong electrical stimulation can induce fibrillation. Electrical defibrillation of the heart by timely application of a strong electric shock is currently the only effective therapy for lethal disturbances in cardiac rhythm. An interesting work was reported by Woltering and Markus [23], who realized suppressing turbulent waves. For an excitable media, they need to apply a finite number of pulses. However, a significant difference is that the final state of the system is

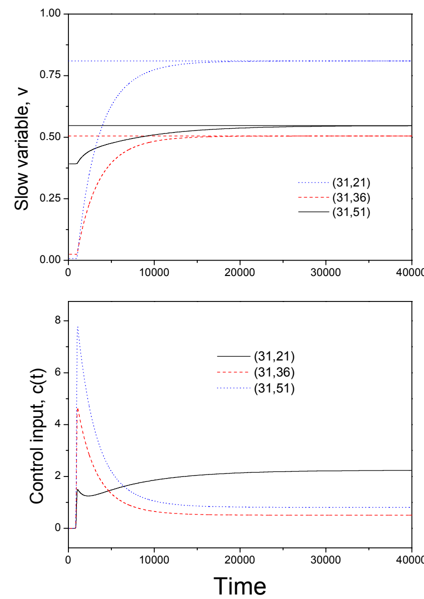

Figure 3: Time evolution of the u, v-field and control input for three positions.

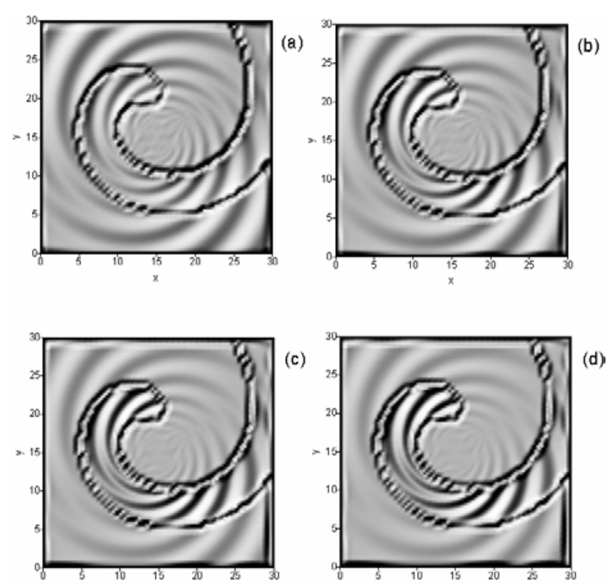

Figure 4: Pattern sequence of control inputs to produce the pattern given in Figure 2-f.

an homogeneous steady-state of the nonlinear waves.

5 Conclusions

In this work, we have presented a feedback control methodology based on modeling error compensation to control spiral waves arising in reaction-diffusion equations describing an excitable media. The control law proposed can be applied to general excitable systems because it is independent of details of the reaction kinetics and model parameters for a large class of models. We have shown via numerical simulations how spiral waves can be controlled to produce new patterns via the introduction of weak external electrical inputs. We hope that our work will be of interest in some important practical applications, such as controlling pattern formation in *D. discoideum,* intracellular waves and ventricular fibrillation in cardiac muscle.

References

[1] Keener, J., Sneyd, J. (1998). *Mathematical Physiology.* Springer-Verlag, Berlin.

[2] Fall, C.P., Marland, E.S., Wagner, J.M., Tyson, J.J. (2002). *Computational Cell Biology.* Springer-Verlag, New York.

[3] Murray, J.D. (1989). *Mathematical Biology.* Springer-Verlag, Berlin.

[4] di Bernardo, D., Signorini, M. R., Cerutti, S. (1998). A model of two nonlinear coupled oscillators for the study of heartbeat dynamics. *Int. J. Bifurcation and Chaos,* **8,** 1975.

[5] Pumir, A., Krinsky, V.I. (1996). How does an electric field defibrillate cardiac muscle?, *Physica* **D91**, 205.

[6] Maini, P.K., Painter, K.J., Phong-Chau, H.N. (1997). Spatial pattern formation in chemical and biological systems. *J. Chem. Soc. Faraday Trans.,* **93**, 3601.

[7] Tyson, J.J. (1985). In: R.J. Field, M. Burger (eds.) *Oscillations and Traveling Waves in Chemical Systems.* Wiley, New York, p. 93.

[8] Barkley, D. (1991). A model for fast computer simulation of waves in excitable media. *Physica D*, **49**, 61.

[9] Zykov, V.S. (1988). *Modeling of Wave Processes in Excitable Media.* Manchester University Press, Princeton.

[10] Belousov, B.P. (1985). In: R.J. Field, M. Burger (eds.) *Oscillations and Traveling Waves in Chemical Systems.* Wiley, New York, p. 605.

[11] Goldbeter, A. (1996). *Biochemical Oscillations and Cellular Rhythms.* Cambridge University Press, Cambridge, UK.

[12] Jakubith, S., Rotermund, H., Engel, W., von Oertzen, A. Ertl, G. (1990). Spatiotemporal concentrations patterns in a surface reaction: propagating and standing waves, rotating spirals, and turbulence, *Phys. Rev. Lett.* **65**, 3013.

[13] Zhang, H. , Zhang, J., Yang, Y., Zhou, X. (1997). Microphase separation of diblock copolymer induced by directional quenching, *J. Chem. Phys.* **106**, 784.

[14] Lu, W., Yu, D., Harrison, R.G. (1996). Control of patterns in spatiotemporal chaos in optics, *Phys. Rev. Lett.* **76**, 3316.

[15] Takeda, S., Koto, K., Iijima, S., Ichihashi, T. (1997). Nanoholes on silicon surface created by electron irradiation under ultrahigh vacuum environment, *Phys. Rev. Lett.* **79**, 2994.

[16] Hu, G., Xiao, J., Chua, L.O., Pivka, L. (1998). Controlling spiral waves in a model of two-dimensional arrays of chua´s circuits, *Phys. Rev. Lett.* **80**, 1884.

[17] Walleczek, J. (2000). The frontiers and challenges of biodynamics research, in *Self-Organized biological dynamics and nonlinear control.* Cambridge University Press, Cambridge, UK.

[18] Zykov, V.S., Mikhailov, A.S., Muller, S.C. (1997). Controlling spiral waves in confined geometries by global feedback, *Phys. Rev. Lett.* **78**, 3398.

[19] Osipov, G.V., Collins, J.J. (1999). Using weak impulses to suppress travelin waves in excitable media, *Phys. Rev. E* **60**, 54-57.

[20] Schebesch, I., Engel, H. (1998). Wave propagation in heterogeneous excitable media, *Phys. Rev. E* **57**, 3905.

[21] Osipov, G.V., Shulgin, B.V., Collins, J.J. (1998). Controlled movement and suppression of spiral waves in excitable media, *Phys. Rev. E* **58**, 6955.

[22] Straughan, B. (1992). *The Energy Method, Stability, and Nonlinear Convection.* Springer-Verlag, New York.

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[23] Christini, D.J., Glass, L. (2002). Introduction: mapping and control of complex cardiac arrhythmias. *Chaos* **12**, 732.

[24] Woltering, M., Markus, M. (2002). Turbulence control by wave splitting in excitable media. *Phys. Lett. A* **297**, 363.

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